

# Exceptional field theories, superparticles in an enlarged 11D superspace and higher spin theories.

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**ABSTRACT:** Recently proposed exceptional field theories (EFTs) making manifest the duality  $E_{n(n)}$  symmetry, first observed as nonlinearly realized symmetries of the maximal  $d = 3, 4, \dots, 9$  supergravity ( $n = 11 - d$ ) and containing 11D and type IIB supergravity as sectors, were formulated in enlarged spacetimes. In the case of  $E_{7(7)}$  EFT such an enlarged spacetime can be identified with the bosonic body of the  $d = 4$  central charge superspace  $\Sigma^{(60|32)}$ , the  $\mathcal{N} = 8$   $d = 4$  superspace completed by 56 additional bosonic coordinates associated to central charges of the maximal  $d = 4$  supersymmetry algebra.

In this paper we show how the hypothesis on the relation of all the known  $E_{n(n)}$  EFTs, including  $n = 8$ , with supersymmetry leads to the conjecture on existence of 11D exceptional field theory living in 11D tensorial central charge superspace  $\Sigma^{(528|32)}$  and underlying all the  $E_{n(n)}$  EFTs with  $n = 2, \dots, 8$ , and probably the double field theory (DFT). We conjecture the possible form of the section conditions of such an 11D EFT and show that quite generic solutions of these can be generated by superparticle models the ground states of which preserve from one half to all but one supersymmetry. The properties of these superparticle models are briefly discussed. We argue that, upon quantization, their quantum states should describe free massless non-conformal higher spin fields in  $D=11$ .

**KEYWORDS:** Supersymmetry, U-duality, superspace, superparticle, higher spin theory .

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## 1. Introduction

Recently exceptional field theories (EFTs), manifestly invariant under U-duality symmetry groups  $E_{n(n)}$  with  $n = 2, 3, 4, 5, 6, 7, 8$  [1] and containing 11D and 10D type IIB supergravity theories as sectors were formulated in enlarged  $d=3,4,5,\dots,9$  spaces [2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19]<sup>1</sup>. The value of  $d$  is related to  $n$  by  $d+n=11$ , and in this sense one can call the  $E_{n(n)}$  EFT ‘ $d$ -dimensional’, the name which also reflects its manifest invariance under the  $d$ -dimensional Lorentz group  $SO(1, d-1)$ <sup>2</sup>. They can be regarded as M-theoretic counterparts of D=10 double field theory (DFT) [22, 23, 24, 25, 26, 28, 27, 29, 30] designed to have a manifest T-duality symmetry, characteristic for string theory<sup>3</sup>. The DFT is formulated in the space with doubled number, 2D, of bosonic coordinates (usually D=10 is assumed in this case). The number of the additional bosonic coordinates  $y^\Sigma$  of the  $d$  dimensional  $E_{n(n)}$  EFT is  $d/n$ - dependent: it varies from 3 in the recently proposed 9d ‘F-theory action’ of [18] to 56 in  $d=4$   $E_{7(7)}$  EFT [7, 9, 11] and 248 in  $d=3$   $E_{8(8)}$  EFT [10, 16]. The dependence of the fields on additional coordinates is restricted by the so-called *section conditions* the strong version of which is imposed (‘by hand’) on any pair of functions of the theory.

$E_{n(n)}$	$n$	$d=11-n$	$N_n = \# \text{ of } y^\Sigma$	Section condition and ref.
$E_{8(8)}$	8	$d=3$	248	$Y_{\Lambda\Sigma}^{\Sigma\Pi} \partial_\Sigma \otimes \partial_\Pi = 0$ , see [10]
$E_{7(7)}$	7	$d=4$	56	$t_G^{\Sigma\Pi} \partial_\Sigma \otimes \partial_\Pi = 0$ , [9], see below
$E_{6(6)}$	6	$d=5$	27	$d^{\Lambda\Sigma\Pi} \partial_\Sigma \otimes \partial_\Pi = 0$ [7]
$E_{5(5)} = SO(5, 5)$	5	$d=6$	16	$\gamma_I^{\Sigma\Pi} \partial_\Sigma \otimes \partial_\Pi = 0$ [15]
$E_{4(4)} = SL(5)$	4	$d=7$	10 ( $y^{ab} = y^{[ab]}$ )	$\partial_{[ab} \otimes \partial_{cd]} = 0$ [3, 6], see below
$E_{3(3)} = SL(3) \times SL(2)$	3	$d=8$	6 ( $y^{\alpha i}$ )	$\epsilon^{ijk} \epsilon^{\alpha\beta} \partial_{\alpha i} \otimes \partial_{\beta j} = 0$ [14]
$E_{2(2)} = SL(2) \times \mathbb{R}^+$	2	$d=9$	3 ( $y^\alpha, z$ )	$\partial_z \otimes \partial_\alpha + \partial_\alpha \otimes \partial_z = 0$ [18]

Table 1. *Additional coordinates and section conditions of the  $E_{n(n)}$  EFTs. The notation for  $n=7$  and  $n=4$  cases are described below. The other cases will not be discussed and we refer to the original papers (cited at the end of the lines) for the notation.*<sup>4</sup>

In the case of DFT the solution of the strong section conditions implies that all the physical fields depend only on D of 2D bosonic coordinates. The manifest T-duality is

<sup>1</sup>The embedding of massive IIA requires a deformation of (section conditions of) the EFT [20, 21].

<sup>2</sup>It is also worth commenting that, while  $E_{6(6)}$ ,  $E_{7(7)}$  and  $E_{8(8)}$  of EFTs with manifest  $d=5,4,3$  Lorentz symmetries are the exceptional Lie groups from the Cartan list, for lower  $n$   $E_{n(n)}$  denote simpler groups:  $E_{5(5)} = SO(5, 5)$ ,  $E_{4(4)} = SL(5)$ ,  $E_{3(3)} = SL(2) \times SL(2)$  and, as it was proposed in recent [18],  $E_{2(2)} = SL(2) \times \mathbb{R}^+$ .

<sup>3</sup>See [31] and refs. therein for T-duality and [32, 33, 34, 35, 36, 30, 37, 38, 39] for string and superstring in doubled (super)spaces. Notice also that we usually denote the number of spacetime dimensions by  $D$  when it is equal to 10 or 11, and by  $d$  when it is lower, so that  $d \leq 9$ .

<sup>4</sup>Notice a partial intersection of (the ‘left hand side’ of) this Table 1 with Table 2 of [40], where a possible relation of 11D supermembrane duality transformations with  $E_{n(n)}$  duality symmetries of dimensionally reduced maximal supergravity was discussed.

provided by the freedom in choosing the set of these  $D$  of the complete set of  $2D$  coordinates. This is called ‘choice of the section’ (hence the name ‘section conditions’ for the equations solved by this choice).

The structure of the EFT section conditions looks strongly  $d$ - (or  $n$ -)dependent and much less transparent. As we will discuss below, the analysis of differences in the structure of EFTs with different  $n$  suggests the possible existence (and makes desirable to find) a hypothetical *underlying EFT*, which we call ‘11D EFT’ or ‘uEFT’, such that all the lower  $d$  EFTs can be obtained by its reductions.<sup>5</sup>

In this paper we make same stages toward the construction of such a hypothetical 11D uEFT. In particular, we argue that the natural basis for its construction is provided by 11D tensorial central charge superspace  $\Sigma^{(528|32)}$ , proposed the section conditions for this uEFT in this superspace, and present a family of superparticle models in  $\Sigma^{(528|32)}$  which produce quite generic solutions of these section conditions. The quantum states of these models are massless, which allows to conjecture that their quantization results in supersymmetric theories of free massless higher spin fields in  $D=11$ . The quantization of  $D=10$  version(s) of the model(s) should produce a theory of free massless non-conformal higher spin field, the tower of which includes 10D ‘graviton’.

The rest of this paper is organized as follows. In the beginning of next Sec. 2 we review the structure of  $E_{n(n)}$  exceptional field theories (EFTs) with  $n = 2, \dots, 8$  and conjecture on their relation with the most general supersymmetry algebra. In particular, in sec. 2.1 we discuss the additional coordinates of  $E_{n(n)}$  EFTs, the section conditions, which are imposed to restrict the dependence of EFT fields on those, and their classical counterparts. In sec. 2.2 we argue in favor of relation of additional coordinates of  $E_{n(n)}$  EFTs with maximal supersymmetry algebra in  $d = 11 - n$ , describe the relation of those with central charges of such a supersymmetry algebra observed first for  $n = 7$ . In sec. 2.3 we discuss the extension of this conjecture to  $n = 8$  which requires involvement of also the vectorial ‘central charges’ and leads us to the most general  $d=3$  maximal supersymmetry algebra.

The underlying EFT (11D EFT or uEFT) conjecture is formulated in Sec. 3. In sec. 3.1 we show that the maximally extended  $d=3$  supersymmetry superalgebra has actually a bigger automorphism symmetry, including  $SO(1, 10)$ , which allows us to call it M-algebra (or M-theory superalgebra), describe the  $SO(1, 10)$  invariant Cartan forms on the associated supergroup manifold  $\Sigma^{(528|32)}$  with 528 bosonic and 32 fermionic directions, and conjecture on the existence of underlying 11D EFT, leaving in this superspace.

In Sec 3.2 we propose the candidate section condition of 11D EFT needed to reduce the huge number of additional bosonic coordinates and discuss the structure of their solutions. In sec. 3.3 we consider a series of superparticle models in  $\Sigma^{(528|32)}$  which produce quite generic solution of the classical section conditions as their constraints. The actions of these models involve essentially 11D spinor moving frame variables [51, 52] (see also [53, 54, 55, 56]), also called Lorentz harmonics [57] (see also [58, 59, 60, 61, 62, 63, 64]); we describe these in sec. 3.3.3.

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<sup>5</sup>Notice that our hypothetical uEFT is not identical but probably complementay to the E11 program of [41, 42, 43, 44, 45, 46, 47, 48, 49, 50]. We will comment on this more in concluding Sec. 5. A discussion on the connection of EFTs and E11 can be found in [47].

In sec. 4 we argue that the quantization of these uEFT-related superparticle models should produce towers of massless 11D higher spin fields as their quantum state spectrum. We briefly describe (in secs. 4.1 and 4.2) the relation of lower dimensional counterpart of preonic superparticle to free massless conformal higher spin fields in  $D = 4, 6, 10$  dimensions and on this basis conjecture (in sec. 4.3) that the counterpart of above mentioned generalized superparticle models with spinor moving frame variables provide the classical mechanic description of massless non-conformal higher spin fields in  $D = 6, 10$ . The quantization of the models in  $\Sigma^{(528|32)}$  should result in a tower of massless non-conformal 11D higher spin fields; the conformal higher spin tower is not known for this case.

We conclude in sec. 5 where the discussion on possible relation/complimentarity of our conjectured 11D EFT and of  $E_{11}$  and  $E_{10}$  hypothesis can be also found.

## 2. On EFTs, their section conditions and central charge superspaces

### 2.1 $E_{n(n)}$ EFTs with $n = 2, \dots, 8$ . Additional coordinates, section conditions and their classical counterparts

Schematically, the  $E_{n(n)}$  EFT is constructed on the basis of maximal  $d = 11 - n$  dimensional supergravity (SUGRA) by allowing the field to depend, besides  $d$  spacetime coordinates  $x^\mu$ , on additional 'internal' coordinates  $y^\Sigma$  the number of which (i.e. the range of the index  $\Sigma$ ), is given by dimension  $N_n$  of minimal irreducible representation of  $E_{n(n)}$  ( $N_2 = 3, \dots, N_7 = 56, N_8 = 248$ ). Besides that, the field strengths and the Lagrangian of  $d$ -dimensional supergravity are modified by inclusion of terms with derivatives  $\partial_\Sigma = \frac{\partial}{\partial y^\Sigma}$ , and the action is constructed by integrating this modified SUGRA Lagrangian  $L_{EFT}^{E_{n,n}}$  over  $d$  spacetime and all the  $N_n$  internal coordinates,  $S_{EFT}^{E_{n,n}} = \int d^4x d^{N_n}y L_{EFT}^{E_{n,n}}$ . This integral is usually considered as formal as far as its rigid definition meets problems related with the next ingredients of EFT which we are going to describe now.

All the fields in EFT,  $F$ , are subject to the so-called weak section conditions

$$Y_{\Lambda\Xi}^{\Sigma\Pi} \partial_\Sigma \partial_\Pi F = 0, \quad (2.1)$$

where  $Y_{\Lambda\Xi}^{\Sigma\Pi}$  is an invariant tensor of  $E_{n(n)}$  the explicit form of which is strongly  $n$ -dependent. But moreover, all the *pairs* of the fields  $F_1, F_2$ , should be subject to the so-called strong section conditions,

$$Y_{\Lambda\Xi}^{\Sigma\Pi} \partial_\Sigma F_1 \partial_\Pi F_2 = 0. \quad (2.2)$$

For  $E_{7(7)}$  EFT, in which  $\Sigma, \Pi, \Lambda, \Xi = 1, \dots, 56$ , these section conditions can be presented in a simpler form

$$t_G^{\Sigma\Pi} \partial_\Sigma F_1 \partial_\Pi F_2 = 0, \quad (2.3)$$

$$\Xi^{\Sigma\Pi} \partial_\Sigma F_1 \partial_\Pi F_2 = 0, \quad (2.4)$$

where  $\Xi^{\Pi\Lambda} = -\Xi^{\Lambda\Pi}$  is the  $Sp(56)$  symplectic 'metric',  $t_G^{\Sigma\Pi} = \Xi^{\Pi\Lambda} t_{G\Lambda}^\Sigma$  and  $t_{G\Lambda}^\Sigma$  are  $E_{7(+7)}$  generators in **56** representation,  $G = 1, \dots, 133$ .

To make the equations lighter, one usually writes the strong and the weak section conditions in the schematic form

$$Y_{\Lambda\Xi}^{\Sigma\Pi}\partial_\Sigma\otimes\partial_\Pi=0\,,\quad (2.5)$$

and

$$Y_{\Lambda\Xi}^{\Sigma\Pi}\partial_\Sigma\partial_\Pi=0\,. \quad (2.6)$$

It is natural to expect that the solutions of the section conditions imply independence of all the fields on some number of internal coordinates. In the above schematic notation this can be expressed as

$$\partial_\Sigma(\dots)=K_\Sigma^r\partial_r(\dots)\,,\quad \partial_r=\frac{\partial}{\partial y^r}\,,\quad r=1,\dots,\tilde{n}_n \quad (2.7)$$

where  $y^r$  are  $\tilde{n}_n$  ( $< N_n$ ) additional coordinates the fields are allowed to depend on. A possible choice of this latter defines a *section* (i.e. a particular solution of the section conditions). The freedom in choosing among the possible sections makes the construction  $E_{n(n)}$ -invariant.

For all the EFTs the  $n$ -parametric and  $(n-1)$ -parametric solutions of the section conditions were found and shown to describe D=11 and D=10 type IIB supergravity [7, 8, 9, 10, 12, 13, 15] (in the majority of the cases the bosonic limit of SUGRA was actually discussed). From the generic String/M-theoretic perspective, one should not expected the possibility to have a solution with functions depending on more than 11 bosonic coordinates. Although for lower  $n$ , e.g. for lowest  $n=2$  case in [18], this is manifest, for higher  $n$  this expectation had been just a reasonable conjecture till recent [65], where it has been proved for the case of d=4  $E_{7(7)}$  EFT. Furthermore, in [65] it was shown that the set of 133 section conditions of this EFT, Eqs. (2.3),

$$t_G^{\Sigma\Pi}\partial_\Sigma\otimes\partial_\Pi=0\,,\quad \Sigma,\Pi=1,\dots,56\,,\quad G=1,\dots,133\,, \quad (2.8)$$

is reducible in the sense that one can extract such a set of 63 conditions that their solution automatically solves also the remaining relations.

To be more specific in this latter statement, it was shown in [65] that the solutions of the set of 63 relations (2.8) involving the generators of  $SU(8)$  subgroup of  $E_{7(+7)}$ ,

$$t_H^{\Sigma\Pi}\partial_\Sigma\otimes\partial_\Pi=0\,,\quad \Sigma,\Pi=1,\dots,56\,,\quad H=1,\dots,63\,, \quad (2.9)$$

automatically solve also the remaining 70 conditions which involve the generators of the coset  $E_{7(+7)}/SU(8)$  ( $t_K^{\Sigma\Pi}\partial_\Sigma\otimes\partial_\Pi=0$ ,  $K=1,\dots,70$ ).

To obtain the above results, it was very useful to analyze the classical mechanic counterpart of the section conditions which reads

$$t_E^{\Sigma\Pi}p_\Sigma p_\Pi=0\,, \quad (2.10)$$

where  $p_\Sigma$  and  $p_\Pi$  are classical momenta of a particle model. One notices that, if we perform a straightforward ‘quantization’ of (2.10) by replacing the momentum by derivative,  $p_\Sigma\mapsto$

$-i\partial_\Sigma$ , consider (2.10) as a (first class) constraint and impose its quantum version as a condition on the wave function, we clearly arrive at the weak version of (2.8) imposed on one function rather than on the pair of functions of EFT. However, as it was discussed in [65], there exists another 'first solve than quantize' way which, starting from the classical section conditions (2.10) results in the (general solution of the) strong section conditions (2.8). The key point is that such a general solution is expected to be of the form of (2.7) and hence can be reproduced by quantization of the general solution of the classical section conditions of the form

$$p_\Sigma = K_\Sigma{}^r p_r, \quad r = 1, \dots, \tilde{n}_n. \quad (2.11)$$

The classical counterpart of the section conditions had been also studied in [82] devoted to development of a twistor approach to  $E_{n(n)}$  EFTs with  $n \leq 6$ . In particular, in [82] it was discussed the classical section conditions of the  $E_{4(4)} = SL(5)$  EFT which reads [3, 6]

$$p_{[ab}p_{cd]} = 0, \quad a, b, c, d = 1, \dots, 5. \quad (2.12)$$

Here  $p_{ab} = -p_{ba}$  are momenta conjugate to the additional bosonic coordinates of the spacetime of the  $E_{4(4)} = SL(5)$  EFT,  $y^{ab} = -y^{ba}$  which belongs to **10** representation of  $SL(5)$ . The simple form of this  $SL(5)$  section conditions will be suggestive for our discussion below.

## 2.2 Some differences between $E_{4(4)}$ , $E_{7(7)}$ and $E_{8(8)}$ EFTs

This is the place to illustrate the differences in the structure of section conditions of  $E_{n(n)}$  EFTs with different  $n$ .

First notice that, as it was shown in [6] the solution of section conditions  $\partial_{[ab} \otimes \partial_{cd]} = 0$  corresponding to the embedding of D=11 and of type IIB supergravity in the d=7  $E_{4(4)}$  EFT are independent in the sense that they are not connected by transformations of  $E_{4(4)} = SL(5)$  group. The same applies to the classical counterparts of this strong section conditions given in Eq. (2.12). In contrast, as it can be deduced from the results of [65], in the case of d=4  $E_{7(7)}$  EFT the situation is opposite: the solutions describing the embedding of D=11 and type IIB supergravities into this EFT are related by transformations of the  $SU(8)$  subgroup of  $E_{7(7)}$ .

Actually this distinction does not look unnatural after comparing the number of bosonic coordinates of  $E_{n(n)}$  EFTs with that of the DFT. Indeed, a unification of the 11D and type IIB solutions of a EFT implies also the unification of (low energy limits of the) type IIA and IIB superstring theories. This is reached in the frame of DFT which is defined in the sapace with doubled number of coordinates,  $2D = 20$ . From this perspective one can expect the independence of 11D and type IIB solution in  $E_{n(n)}$  EFTs with  $2 \leq n \leq 4$ , where the number of additional and spacetime coordinates is less than 20, and their unification in EFTs with  $n \geq 5$ . It will be interesting to check this hypothesis for  $n = 5, 6$  and  $n = 8$  cases.

One more illustrative example is in difference between  $E_{7(7)}$  and  $E_{8(8)}$  EFTs. The first is formulated in the space with 56 additional coordinates  $y^\Sigma = (y^{ij}, \bar{y}_{ij})$  which can

be considered [65] as a bosonic body of central charge superspace  $\Sigma^{(60|32)}$  [66]<sup>6</sup>. The flat version of this superspace,  $\Sigma_0^{(60|32)}$ , is the supergroup manifold associated with the most general central extension of the maximal  $D = 4$   $\mathcal{N} = 8$  supersymmetry algebra

$$\{Q_\alpha^i, Q_\beta^j\} = \epsilon_{\alpha\beta} Z^{ij}, \quad \{Q_\alpha^i, \bar{Q}_{\dot{\beta}j}\} = \delta_j^i \sigma_{\alpha\dot{\beta}}^a P_a, \quad \{\bar{Q}_{\dot{\alpha}i}, \bar{Q}_{\dot{\beta}j}\} = \epsilon_{\dot{\alpha}\dot{\beta}} \bar{Z}_{ij}, \quad (2.13)$$

$$\alpha, \beta = 1, 2, \quad \dot{\alpha}, \dot{\beta} = 1, 2, \quad a = 0, 1, 2, 3, \quad i, j = 1, \dots, 8.$$

This observation allowed us [65] to formulate a superparticle model in central charge superspace which generates the classical counterpart of the independent section conditions (2.9) as a constraint. The model is an improved version of the  $\mathcal{N} = 8$  superparticle described by de Azcárraga and Lukiersi in [83]. In the original model the invariance of the action under  $\kappa$ -symmetry can be reached only if we allow ourselves to impose the classical counterpart of independent section conditions, (2.10) with  $E = H$ ,

$$t_H^{\Sigma\P} p_\Sigma p_\Pi = 0, \quad H = 1, \dots, 63, \quad (2.14)$$

‘by hand’, while in our improved version (which is not apparently equivalent to the original model) these appear as equations of motion.

A natural wish is to find a similar superparticle model generating (an independent part of) the section conditions for  $E_{8(8)}$  EFT. But here we meet a problem already at the first stage. The number of central charges  $Z^{pq} = -Z^{qp}$  of the central extension of maximal  $d = 3$  supersymmetry algebra

$$\{Q_{\tilde{\alpha}}^q, Q_{\tilde{\beta}}^p\} = \gamma_{\tilde{\alpha}\tilde{\beta}}^{\tilde{a}} \delta^{pq} P_{\tilde{a}} + i \epsilon_{\tilde{\alpha}\tilde{\beta}} Z^{pq}, \quad \tilde{\alpha}, \tilde{\beta} = 1, 2, \quad \tilde{a} = 0, 1, 2, \quad p, q = 1, \dots, 16 \quad (2.15)$$

is 120, while the number of the additional coordinates of  $E_{8(8)}$  EFT is 248 [7]. This is the dimension of the minimal irreducible representation of  $E_{8(8)}$  which in [7] was taken to be the adjoint representation.

Thus the relation of additional coordinates of EFT with central charge of maximal central extension of the maximal  $d$ -dimensional supersymmetry algebra observed for  $E_{7(7)}$  EFT in [65] cannot be generalized straightforwardly to the  $E_8$  case.

The idea of our study is to insist nevertheless on the beautiful relation of additional coordinates of EFT and of the maximal  $d$  dimensional supersymmetry algebra. As we will see in a moment, this leads us to the conjecture on the existence of an underlying EFT (uEFT) ‘living’ in the maximal *tensorial* central charge superspace. This can be defined at any  $d \leq 11$ , but its associated supersymmetry algebra always has a hidden symmetry including  $SO(1, 10)$  so that it can be called M-algebra or M-theory superalgebra and our uEFT can be called 11D EFT.

### 2.3 $E_{8(8)}$ EFT and maximal supersymmetry

If we insist on relation of additional coordinates of the  $E_{n(n)}$  EFT with maximal  $d = 11 - n$  supersymmetry algebra, in the case of  $n = 8$ ,  $d = 3$  we have to allow for contributions

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<sup>6</sup>The formulation of standard  $\mathcal{N} = 8$   $d = 4$  supergravity in this superspace was described in (Appendix B of) [66] and in (Appendix C of) more recent [67], where the central charge superspace formulation of other maximal  $d=11-n$  supergravities with  $2 \leq n \leq 8$  were also considered.



of some additional coordinates carrying both the indices of the internal symmetry  $SO(16)$  and of the  $d=3$  Lorentz symmetry. Namely, we need in a coordinates conjugate to 128 of possible 405 additional vectorial 'central' charges  $Y_a^{pq} = Y_a^{((pq))}$  (where double brackets imply symmetric traceless part:  $Y_a^{pq} = Y_a^{qp}$ ,  $Y_a^{qq} = 0$ ).

With the contribution of all these generators the right hand side (*r.h.s.*) of the defining relation of the maximal  $d = 3$  supersymmetry algebra,

$$\{Q_{\tilde{\alpha}}^q, Q_{\tilde{\beta}}^p\} = \gamma_{\tilde{\alpha}\tilde{\beta}}^{\tilde{a}}(\delta^{pq}P_{\tilde{a}} + Y_{\tilde{a}}^{((pq))}) + i\epsilon_{\tilde{\alpha}\tilde{\beta}}Z^{pq}, \quad (2.16)$$

$$\tilde{\alpha}, \tilde{\beta} = 1, 2, \quad \tilde{a} = 0, 1, 2, \quad p, q = 1, \dots, 16$$

becomes the generic 528 component  $32 \times 32$  matrix ( $528=3+405+120$ ).

The mechanism of extraction of  $128(=248-120)$  additional coordinates of  $E_{8(8)}$  EFT of  $405(=528-3-120)$  additional coordinates conjugate to the vectorial central charge of (2.16) should be dynamical. A search for it is beyond the scope of this paper. For our discussion here the presence of even more ('beyond the  $E_{8(8)}$  EFT') additional coordinates is not problematic but rather suggestive.

Indeed at this stage it is tempting to conjecture the existence of an underlying exceptional field theory (uEFT), which includes as a sub-sectors all the  $E_{n(n)}$  EFT with  $2 \leq n \leq 8$  and lives in an enlarged superspace  $\Sigma^{(528|32)}$  with 32 fermionic and 528 bosonic coordinates. In terms of the above discussed  $n = 8$  case, these latter can be split on  $d = 11 - n = 3$  spacetime, 120 central charge and 405 'vector central charge' coordinates. But actually the similar splitting is possible for any  $n$ : the number of spacetime coordinates will be  $d = 11 - n$  while the set of additional  $528 - d$  coordinates will be split, in an  $SO(1, d - 1)$  invariant way, on the subsets of scalar, vector and tensorial 'central' charge coordinates. The reason beyond this lays in a huge hidden automorphism symmetry of the superalgebra (2.16) which we are going to discuss now.

### 3. Maximal 11D tensorial central charge superspace $\Sigma^{(528|32)}$ and uEFT conjecture

#### 3.1 $\Sigma^{(528|32)}$ geometry and maximal supersymmetry

The manifest  $SO(1, 2) \times SO(16)$  symmetry of (2.16) is related to the basis we have used to decompose the matrix of generators in *r.h.s.* of this relation. There exists also the manifestly  $SO(1, 10)$  invariant form of the same relation,

$$\{Q_{\alpha}, Q_{\beta}\} = i\Gamma_{\alpha\beta}^a P_a + \Gamma_{\alpha\beta}^{ab} Z_{ab} + i\Gamma_{\alpha\beta}^{abcde} Z_{abcde}, \quad (3.1)$$

$$a, b, c = 0, \dots, 9, 10, \quad \alpha, \beta, \gamma = 1, \dots, 32.$$

which explains the name of *M-algebra* or *M-theory superalgebra* often used for this most general supersymmetry superalgebra<sup>7</sup>. The generators  $Z_{ab} = Z_{[ab]}$  and  $Z_{abcde} = Z_{[abcde]}$  are called *tensorial central charges*.

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<sup>7</sup>Two comments are in time. Firstly, the algebra (3.1) was described much before the M-theory epoch in [68] and [69]. Secondly, in [84] the name 'M-algebra' was used for the superalgebra with additional fermionic generators.

Actually, the M-algebra possesses  $GL(32)$  automorphism symmetry which becomes manifest if we write it in the form

$$\{Q_\alpha, Q_\beta\} = i\mathcal{P}_{\alpha\beta}, \quad \alpha, \beta = 1, 2, \dots, 32, \quad (3.2)$$

collecting all the generators in the *r.h.s.* in one symmetric  $32 \times 32$  matrix  $\mathcal{P}_{\alpha\beta}$  ( $528 = \frac{32 \times 33}{2}$ ). Decomposing this on the basis of 11D gamma matrices and their products,

$$\mathcal{P}_{\alpha\beta} = \Gamma_{\alpha\beta}^a P_a - i\Gamma_{\alpha\beta}^{ab} Z_{ab} + \Gamma_{\alpha\beta}^{abcde} Z_{abcde}, \quad (3.3)$$

we arrive at the form (3.1) of the M-algebra, in which only the  $SO(1,10)$  symmetry is manifest. The transformations from the  $GL(32)/SO(1,10)$  coset mixes the vector and antisymmetric tensor central charges among themselves.

If we complete the D=11 superspace by introduce the coordinates dual to every tensorial central charge generator, we arrive at superspace  $\Sigma^{(528|32)}$  which is the supergroup manifold corresponding to the maximal supersymmetry algebra (3.2) or (3.1). We denote coordinates of this superspace by

$$\mathcal{Z}^{\mathfrak{M}} = (X^{\alpha\beta}, \theta^\alpha) = (x^a, y^{ab}, y^{abcde}, \theta^\alpha), \quad (3.4)$$

$$X^{\alpha\beta} = X^{\beta\alpha} = \frac{1}{32} x^a \tilde{\Gamma}_a^{\alpha\beta} - \frac{i}{64} y^{ab} \tilde{\Gamma}_{ab}^{\alpha\beta} + \frac{1}{32 \cdot 5!} y^{abcde} \Gamma_{\alpha\beta}^{abcde}. \quad (3.5)$$

The supersymmetric invariant Cartan forms of  $\Sigma^{(528|32)}$  can be collected in a simple expressions

$$\Pi^{\alpha\beta} = dX^{\alpha\beta} - id\theta^{(\alpha} \theta^{\beta)}, \quad \Pi^\alpha = d\theta^\alpha, \quad (3.6)$$

which are covariant under  $GL(32)$ . The  $SO(1,10)$  invariant decomposition of the bosonic form reads  $528=11+55+462$ , i.e. (see [75] for properties of 11D gamma matrices in our notation)

$$\Pi^{\alpha\beta} = \frac{1}{32} \Pi^a \tilde{\Gamma}_a^{\alpha\beta} - \frac{i}{64} \Pi^{ab} \tilde{\Gamma}_{ab}^{\alpha\beta} + \frac{1}{32 \cdot 5!} \Pi^{abcde} \Gamma_{\alpha\beta}^{abcde}, \quad (3.7)$$

where

$$\begin{aligned} \Pi^a &= dx^a - id\theta^\alpha \Gamma_{\alpha\beta}^a \theta^\beta, & \Pi^{ab} &= dy^{ab} - d\theta^\alpha \Gamma_{\alpha\beta}^{ab} \theta^\beta, \\ \Pi^{abcde} &= dy^{abcde} - id\theta^\alpha \Gamma_{\alpha\beta}^{abcde} \theta^\beta. \end{aligned} \quad (3.8)$$

Our discussion above suggests to try to use the curved D=11 tensorial central charge superspace  $\Sigma^{(528|32)}$  as an arena for constructing the *11D EFT*, underlying the 'd = 11 - n dimensional'  $E_{n(n)}$  EFTs with  $n \leq 8$  (hence the name *uEFT* which we also use for this 11D EFT).

The above described enlarged 11D superspace  $\Sigma^{(528|32)}$  with additional tensorial central charge coordinates,  $y^{ab}$  and  $y^{abcde}$  in (3.5), was discussed in different contexts in [69, 70, 40, 71, 72, 73, 74, 75, 76]. Of course, its 528 bosonic coordinates can be considered as finite subset of the infinite set of tensorial coordinates which were introduced in [42] in the frame

of  $E_{11}$  proposal [41]–[50] (see concluding section 5 for more discussion on this). Notice also the relation of  $\Sigma^{(528|32)}$  with hidden gauge symmetry [68, 74, 75] of 11D supergravity <sup>8</sup> [85], and that in this context the  $GL(32)$  symmetry of  $\Sigma^{(528|32)}$  is also broken down to its  $O(1, 10)$  subgroup.

The useful fact for our discussion below is that the  $D = 4, 6$  and 10 counterparts of this maximally enlarged superspace,  $\Sigma^{(m(m+1)/2|m)}$  with  $m = 4, 8, 16$ , provide the arenas for constructing free massless conformal higher spin theories in  $D = 4, 6, 10$  dimensional spacetimes [86, 87, 88, 90, 91, 92, 93, 94, 95, 96], and that these theories do possess  $GL(n)$  and, moreover, the generalized superconformal  $OSp(1|2n)$  symmetries.

We appreciate that the relation  $d = 11 - n$  might suggest  $d = 1$  or  $d = 0$  EFT to be the underlying one. However, such hypothetical EFTs should have infinite dimensional symmetry groups  $E_{10}$  [77, 78, 79, 80, 81] and  $E_{11}$  [41, 42, 43, 44, 45, 46, 47, 48, 49, 50], which seems to imply the necessity to introduce an infinite number of additional coordinates. In contrast a huge but finite number of unconventional coordinates in our 11D EFT (528) provides us with the resource for additional coordinates for all the  $E_{n(n)}$  EFTs with  $2 \leq n \leq 8$  (and also for 10D DFT) although do not make any of these U-duality symmetries manifest. More discussion on possible relation/complimentarity of our 11D EFT and  $E_{11}$  proposal can be found in Sec. 5. In the next section we present the possible section conditions of the hypothetical 11D EFT.

### 3.2 Section conditions of the hypothetical 11D EFT

In this section we propose the set of section conditions which can be used to reduce the number of spacetime coordinates in the hypothetical 11D EFT.

The proposed set of additional coordinates of the hypothetical uEFT,  $y^{ab}$  and  $y^{abcde}$ , resembles the variables  $y^{ab} = -y^{ba}$  of the  $E_{4(4)} = SL(5)$  EFT, with the evident difference that in our case antisymmetric tensor coordinate carry Lorentz group indices, the same as the usual vector coordinate  $x^a$ ,  $a = 0, \dots, 9, 10$ . Then the simple form of the section conditions for  $E_{4(4)} = SL(5)$  EFT [3, 6], Eq. (2.12), suggests to try the following candidate section conditions for the hypothetical 11D uEFT:

$$\partial_{[a_1 \dots a_k} \otimes \partial_{b_1 \dots b_l]} + \partial_{[b_1 \dots b_l} \otimes \partial_{a_1 \dots a_k]} = 0, \quad k, l = 1, 2, 5. \quad (3.9)$$

The classical mechanic counterparts of these relations are

$$p_{[a} p_{bc]} = 0, \quad p_{[a} p_{bcdef]} = 0, \quad (3.10)$$

$$p_{[ab} p_{cd]} = 0, \quad p_{[ab} p_{c_1 \dots c_5]} = 0. \quad (3.11)$$

One might want to add  $p_{[a} p_{b]} = 0$  and  $p_{[b_1 \dots b_5} p_{c_1 \dots c_5]} = 0$ , but these are satisfied identically at the classical level.

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<sup>8</sup>In [40] the coordinates  $y^{ab}$  were introduced to describe the duality transformations of supermembrane, and their possible relation with  $E_{n(n)}$  duality symmetries was discussed. In this respect it looks interesting that, as it was found in [74, 75], the hidden gauge symmetry of 11D supergravity can be associated with a one-parametric family of superalgebras the associated supergroup manifold of which generically includes  $\Sigma^{(528|32)}$ , but one of the elements of this family is associated with a smaller enlarged 11D superspace, containing  $y^{ab}$ , but not  $y^{abcde}$  coordinate.

The trivial solution of these section conditions,  $p_{ab} = 0 = p_{c_1 \dots c_5}$ , should reduce the uEFT to 11D supergravity. We expect also to have solutions which correspond to embedding of  $E_{n(n)}$  ETS with  $n \leq 8$ .

Actually it is not difficult to find the general solution of the first two equations, (3.10). It reads

$$p_{ab} = p_{[a}q_{b]} , \quad p_{abcde} = p_{[a}q_{bcde]} , \quad (3.12)$$

with arbitrary  $q_b$  and  $q_{bcde} = q_{[bcde]}$ . This solves also the remaining part of the classical section conditions, (3.11).

The easiest way to impose this solution of the section conditions on a function on the bosonic bosonic body  $\Sigma^{(528|0)}$  of  $\Sigma^{(528|32)}$  (i.e. to quantize the classical section conditions using 'first solve then quantize' method), passes through the Fourier transform with respect to  $x^a$ . The quantum version of (3.12) imposed on the (wave)function  $\Phi(p_c, y^{cd}, y^{cdefg}) \equiv \Phi(p, y^{[2]}, y^{[5]})$ ,

$$\begin{aligned} \partial_{ab}\Phi(p, y^{[2]}, y^{[5]}) &= -ip_{[a}q_{b]}\Phi(p, y^{[2]}, y^{[5]}), \\ \partial_{abcde}\Phi(p, y^{[2]}, y^{[5]}) &= -ip_{[a}q_{bcde]}\Phi(p, y^{[2]}, y^{[5]}), \end{aligned} \quad (3.13)$$

is solved by

$$\Phi(p, y^{[2]}, y^{[5]}) = \exp\{-iy^{bc}p_{[b}q_{c]} - iy^{bcdef}p_{[b}q_{cdef]}\} \phi(p_a, q_a, q_{a_1 a_2 a_3 a_4}). \quad (3.14)$$

One can appreciate that, as it is defined in the above equations,  $\phi(p_a, q_a, q_{a_1 a_2 a_3 a_4})$  dependence on  $q_a$  and  $q_{abcd}$  should be such that the following redefinition of these do not change  $\phi(p_a, q_a, q_{a_1 a_2 a_3 a_4})$  :

$$q_a \sim q_a + \tilde{q}p_a , \quad q_{abcd} \sim q_{abcd} + \tilde{q}_{[bcd}p_{a]} . \quad (3.15)$$

Taking into account that in this equations the second 'symmetry' is reducible,

$$\tilde{q}_{abc} \sim \tilde{q}_{abc} + \tilde{\tilde{q}}_{[ab}p_{c]} , \quad \tilde{\tilde{q}}_{ab} \sim \tilde{\tilde{q}}_{ab} + \tilde{\tilde{q}}_{[a}p_{b]} , \quad (3.16)$$

one finds that effectively  $\phi(p_a, q_a, q_{a_1 a_2 a_3 a_4})$  depends on  $11 - 1 + \{\frac{11}{4}\} - \{\frac{11}{3}\} + \{\frac{11}{2}\} - \{\frac{11}{1}\} = 219$  additional momenta. On first glance this might look damaging for our hypothesis, as 219 is clearly less than 248, the dimension of minimal irreducible representation of  $E_{8(8)}$ . However, let us recall that also in the case of  $E_{8(8)}$  EFT one expects the general solution of its section condition to allow dependence of the functions on not more than 7 coordinates, while a dependence on other 241 coordinates is 'unphysical' but needed to provide a freedom in choosing section and thus the  $E_8$  invariance.

This suggests that the way from uEFT to  $E_{8(8)}$  EFT might pass through generic function  $\phi(p_a, q_a, q_{a_1 a_2 a_3 a_4})$ , depending in an arbitrary ('unphysical') manner on  $11 + 330 = 341 > 248$  variables  $q_a$  and  $q_{a_1 a_2 a_3 a_4}$ , and assume that, at the intermediate stage, an independence on some part of these coordinates appears due to some (dynamical or imposed) reduction mechanism. In this paper we will not try to find such a mechanism but rather

assume its existence and exploit further the consequence of the idea of possible existence of the 11D uEFT.

Below we will describe a set of superparticle models proposed in [72] and show that these produce quite generic solutions of the above section conditions as equations of motion. One of these models possess the maximal number 32 of supersymmetries and 31 local fermionic  $\kappa$ -symmetries so that it has a properties of BPS preon [73]. It is nevertheless different from the original 'preonic superparticle' of [71], and this difference results in breaking of the generalized superconformal symmetry  $OSp(1|64)$  characteristic for the model of [71].

### 3.3 Dynamical model generating (solutions of) the section conditions

Having a candidate set of section conditions, first questions to answer is whether the corresponding EFT subject to this conditions has nontrivial solutions and, if yes, whether these are meaningful in the perspective of String/M-theory. In this sec. 3.3 we address a classical counterpart of the first of these problems: we search for supersymmetric particle models in  $\Sigma^{(528|32)}$  generating solutions of the classical section conditions (3.10), (3.11). The meaning of these models will be the subject of the next Sec. 4.

#### 3.3.1 Preonic superparticle and conformally invariant section conditions

The most known superparticle model in maximal tensorial central charge superspace is the 'preonic superparticle' of [71]. Its action

$$S = \int d\tau \lambda_\alpha \lambda_\beta \Pi_\tau^{\alpha\beta} \equiv \int d\tau \lambda_\alpha \lambda_\beta (\partial_\tau X^{\alpha\beta} - i \partial_\tau \theta^{(\alpha} \theta^{\beta)}) \quad (3.17)$$

contains, besides the bosonic and fermionic coordinate functions,  $X^{\alpha\beta}(\tau) = X^{\beta\alpha}(\tau)$  and  $\theta^\alpha(\tau)$ , also independent bosonic spinor field  $\lambda_\alpha(\tau)$ ,  $\alpha = 1, \dots, 32$ .

Actually the model can be defined with arbitrary number  $m$  of values of the indices,  $\alpha, \beta = 1, \dots, m$ , and for each value of  $m$  it possesses a rigid symmetry under  $OSp(1|2m)$  supergroup as well as local  $\frac{m(m-1)}{2}$  parametric bosonic symmetry ( $b$ -symmetry) and local  $(m-1)$  parametric fermionic  $\kappa$ -symmetry [71]. For  $m = 4, 8, 16$  cases  $\alpha, \beta$  can be treated as  $Spin(1, D-1)$  indices (i.e.  $SO(1, D-1)$  spinor indices) of  $D = 4, 6, 10$  dimensional spacetime and the quantization of the corresponding model results in an infinite tower of free conformal higher spin fields in these dimensions [86, 92]. The role of the generalized superconformal group is played in this approach by  $OSp(1|2m)$  supergroup with the bosonic body  $Sp(2m)$  playing the role of generalized conformal group [97, 86, 87, 88]

For our original case of  $m = 32$ , the action possesses 31  $\kappa$ -symmetries and this implies that the ground state of the model preserves all but one 11D spacetime supersymmetry i.e. possess the property of BPS preon of M-theory in the terminology of [73] (see [76] for a review). This is the reason to apply (*a posteriori*) the name 'preonic superparticle' to the model of [71]. However, the quantization of the  $m = 32$  ( $D = 11$ ) preonic superparticle results in a quantum state spectrum including state vectors with an indefinite mass. The physical interpretation of such quantum states is obscure.

For the generic value of  $m$  the ground state of the model (3.17) preserves  $(m - 1)$  of  $m$  supersymmetries of  $\Sigma^{(\frac{m(m+1)}{2}|m)}$  superspace which allows us to apply the name ‘preonic superparticle’ also to the cases of  $m = 2, 4, 8, 16$  when the treatment of  $\alpha, \beta$  as spinor indices is possible.

The canonical momentum conjugate to the bosonic coordinate function of the preonic superparticle,  $p_{\alpha\beta} := \frac{\partial L}{\partial \partial_\tau X^{\alpha\beta}}$ , is expressed through the bilinear of the bosonic spinors,

$$p_{\alpha\beta} = \lambda_\alpha \lambda_\beta . \quad (3.18)$$

This provides a general solution of a kind of  $GL(n)$  invariant counterpart of the classical section conditions:

$$p_{\alpha[\beta} p_{\gamma]\delta} = 0 . \quad (3.19)$$

The corresponding counterpart of weak section condition imposed on a (wave) function

$$\partial_{\alpha[\beta} \partial_{\gamma]\delta} \phi(X) = 0 \quad (3.20)$$

gives the bosonic equation proposed by Vasiliev in [87, 88]. For  $n = 4, 8, 16$  the solutions of this equation describe the tower of free massless bosonic conformal higher spin fields in  $D = 4, 6, 10$  (see [92] for  $D=6, 10$  cases).

However, the fact that for  $m = 32$  the meaning of this equation and of its solutions is unclear defends us from temptation to propose its ‘strong’ generalization

$$\partial_{\alpha[\beta} \otimes \partial_{\gamma]\delta} + \partial_{\delta[\gamma} \otimes \partial_{\beta]\delta} = 0 \quad (3.21)$$

as a candidate strong section condition for our hypothetical uEFT <sup>9</sup>.

Decomposing the symmetric  $32 \times 32$  matrix of the generalized momentum (3.18) of the preonic superparticle model on the basis of 11D Dirac matrices (see (3.5)), we find the corresponding vector momentum and its tensor counterparts read

$$p_a = \lambda \tilde{\Gamma}_a \lambda , \quad p_{ab} = i \lambda \tilde{\Gamma}_{ab} \lambda , \quad p_{abcde} = \lambda \tilde{\Gamma}_{abcde} \lambda . \quad (3.22)$$

For the generic bosonic spinor  $\lambda_\alpha$  these do not obey the relation (3.10) and (3.11) which we have proposed as candidate section conditions for the hypothetical 11D uEFT. Thus we have to search for a different  $\Sigma^{(528|32)}$  superparticle model to generate (solutions of) these.

### 3.3.2 Preonic superparticle with composite bosonic spinor

Curiously enough, a simple modification of the preonic superparticle model makes it to obey the proposed classical section conditions of uEFT, (3.10) and (3.11). To this end it is sufficient to make the fundamental bosonic spinor  $\lambda_\alpha$  composite,

$$\lambda_\alpha = \lambda_q^+ v_\alpha^{-q} . \quad (3.23)$$

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<sup>9</sup>Notice that the solutions of Eq. (3.20), the form of which can be found in which in sec. 4.2, also solve the ‘strong’ condition (3.21). This is a good illustration of the statement (in sec. 2.1) that ‘first solve than quantize’ approach provides a solution of strong section conditions.

Here  $\lambda_q$  is a 16 component bosonic vector (spinor of  $SO(9)$ ),  $q = 1, \dots, 16$ , and  $v_{\alpha q}^-$  is a set of 16 Majorana spinors of  $SO(1, 10)$  ( $\alpha = 1, \dots, 32$ ) constrained by (see [51, 52] and below for more details and references)

$$2v_{\alpha}^{-q}v_{\beta}^{-q} = u_a^- \Gamma_{\alpha\beta}^a, \quad v_{\alpha}^{-q}\tilde{\Gamma}_a^{\alpha\beta}v_{\beta}^{-p} = \delta^{qp}u_a^-, \quad v_{\alpha}^{-q}C^{\alpha\beta}v_{\beta}^{-p} = 0. \quad (3.24)$$

These constraints include the 11D gamma matrices  $\Gamma_{\alpha}^{a\beta}$  and charge conjugation matrix  $C^{\alpha\beta}$ ; they are both imaginary in our mostly minus notation while

$$\Gamma_{\alpha\beta}^a = \Gamma_a^{\alpha\gamma}C_{\gamma\beta}, \quad \tilde{\Gamma}_a^{\alpha\beta} = C^{\alpha\gamma}\Gamma_{\gamma}^{a\beta} \quad (3.25)$$

are real and symmetric.

The sign indices of spinors,  $\pm$ , and vectors,  $\#$  ( $=^{++}$ ) and  $=$  ( $=^{--}$ ), indicate the weight of different variables under  $SO(1, 1)$  transformations which play an important role when clarifying the group theoretical meaning of the constrained variables  $v_{\alpha q}^-$ ; we will discuss this in the next section.

It is important that the constraints (3.24) imply that the vector  $u_a^-$  is light-like,

$$u^{=a}u_a^- = 0. \quad (3.26)$$

Furthermore, we can show that, as a result of these constraints, both the spacetime momentum  $p_a$  and the momenta conjugate to the tensorial central charge coordinates,  $p_{ab}$  and  $p_{abcde}$ , are proportional to  $u_a^-$ ,

$$p_a = u_a^- \rho^{\#}, \quad (3.27)$$

$$p_{ab} = u_{[a}^- q_{b]}^{\#}, \quad p_{abcde} = u_{[a}^- q_{bcde]}^{\#}, \quad (3.28)$$

where

$$\rho^{\#} = \lambda_q^+ \lambda_q^+, \quad (3.29)$$

and  $q_b^{\#}$  and  $q_{bcde}^{\#}$  are also certain bilinears of  $\lambda_q^+$  (we describe them below). It is easy to see that (3.27), (3.28) solve the candidate uEFT section conditions (3.10) and (3.11).

Thus we have shown that a solution of the candidate section conditions (3.10) and (3.11) is generated by the generalized superparticle model with the action

$$S = \int d\tau \lambda_q^+ v_{\alpha}^{-q} \lambda_p^+ v_{\beta}^{-p} \Pi_{\tau}^{\alpha\beta} \equiv \int d\tau \lambda_q^+ \lambda_p^+ v_{\alpha}^{-q} v_{\beta}^{-p} (\partial_{\tau} X^{\alpha\beta} - i\partial_{\tau} \theta^{(\alpha} \theta^{\beta)}) \quad (3.30)$$

where  $X^{\alpha\beta}(\tau)$  and  $\theta^{\alpha}(\tau)$  are bosonic and fermionic coordinate functions describing the embedding of the superparticle worldline into 11D tensorial central charge superspace  $\Sigma^{(528|32)}$ ,  $\lambda_q^+(\tau)$  are 16-component bosonic vectors (which can be considered as spinors of  $SO(9)$ ) and  $v_{\alpha}^{-q}(\tau)$  is a set of bosonic variables constrained by (3.24).

### 3.3.3 Spinor moving frame variables

We have seen that the constraints (3.24) are useful as due to them the momenta conjugate to the 11D spacetime coordinate and to the tensorial central charge coordinates obey the classical section conditions (3.10) and (3.11). Furthermore, they also imply that the projection of the worldline to 11D spacetime is light-like as (3.24) result in (3.27), (3.26) and, hence,

$$p^a p_a = 0 . \quad (3.31)$$

This implies that the quantum states of our dynamical systems are massless from the perspective of 11D spacetime.

However, at first glance, the meaning of the constraints (3.24) might look obscure. To clarify this, let us first notice that the above constraints have a trivial solution

$$v_\alpha^{-q} = \delta_\alpha^q = \begin{pmatrix} 0 & 0 \\ 0 & I_{16 \times 16} \end{pmatrix} \quad (3.32)$$

for which (with an appropriate representation of the 11D gamma matrices)  $u_a^{=(0)} = \delta_a^0 - \delta_a^{10}$  and the composed bosonic spinor (3.23) has 16 vanishing components,

$$\lambda_\alpha^0 = \lambda_q^+ \delta_\alpha^q = \begin{pmatrix} 0 & 0 & \dots & 0 & \lambda_1^+ & \lambda_2^+ & \dots & \lambda_{16}^+ \end{pmatrix} . \quad (3.33)$$

The solution (3.32) breaks the manifest  $SO(1,10)$  Lorentz symmetry of the constraints (3.24) to its  $[SO(1,1) \otimes SO(9)] \otimes \mathbb{K}_9$  subgroup (see below for definition of  $\mathbb{K}_9$ ). The general solution is given by a Lorentz rotated version of (3.33) in which the parameters of the Lorentz rotations are considered as additional dynamical variables.

A Lorentz rotation of 11D spinors are described by real  $32 \times 32$  matrix taking values in the double covering of the 11D Lorentz group,  $Spin(1,10)$ ,

$$V_\alpha^{(\beta)} = \left( v_\alpha^{+q}, v_\alpha^{-q} \right) \in Spin(1,10) , \quad \alpha = 1, \dots, 32 , \quad q = 1, \dots, 16 . \quad (3.34)$$

When the elements of this matrix is considered as fields, in our case as 1-dimensional fields  $V_\alpha^{(\beta)}(\tau) = \left( v_\alpha^{+q}(\tau), v_\alpha^{-q}(\tau) \right)$ , (3.34) can be called *spinor moving frame matrix*. This matrix and its counterpart with sign inverted,  $-V_\alpha^{(\beta)}(\tau)$ , are in two-to-one correspondence with the *moving frame matrix*, which is  $SO(1,10)$  valued matrix  $U_a^{(b)}(\tau)$

$$U_a^{(b)} = \left( \frac{1}{2} \left( u_a^- + u_a^\# \right), u_a^I, \frac{1}{2} \left( u_a^\# - u_a^- \right) \right) \in SO(1, D-1) ; \quad (3.35)$$

this describes the moving frame attached to the worldline.

The correspondence is given by the conditions of Lorentz invariance of the Gamma matrices (see (3.25))

$$V \Gamma^{(a)} V^T = \Gamma^b U_b^{(a)} , \quad (3.36)$$

$$V^T \tilde{\Gamma}_b V = U_b^{(a)} \tilde{\Gamma}_{(a)} , \quad (3.37)$$



and of the charge conjugation matrix

$$VCV^T = C. \quad (3.38)$$

The splitting of the Lorentz group valued matrix  $U_a^{(b)}$  in (3.35) is invariant under  $SO(1,1) \otimes SO(9)$  subgroup of  $SO(1,10)$ , and the condition  $U_a^{(b)} \in SO(1,10)$  implies the following conditions on the vectors forming this matrix (see [58, 59])

$$u_a^- u^a = 0, \quad u_a^- u^{aI} = 0, \quad u_a^- u^{a\#} = 2, \quad (3.39)$$

$$u_a^\# u^{a\#} = 0, \quad u_a^\# u^{aI} = 0, \quad (3.40)$$

$$u_a^I u^{aJ} = -\delta^{IJ}. \quad (3.41)$$

With the suitable representation for 11D gamma matrices, the conditions of correspondence between moving frame and spinor moving frame variables, (3.36), (3.37) and (3.38) (equivalent to  $V_\alpha^{(\beta)} \in Spin(1,10)$  and actually defining  $U_a^{(b)} \in SO(1,10)$ ) can be split into the following set of constraints for the spinor moving frame variables (this is to say for the rectangular blocks of  $V_\alpha^{(\beta)} \in Spin(1,10)$ )

$$2v_\alpha^{-q} v_\beta^{-q} = \Gamma_{\alpha\beta}^a u_a^- \quad (a), \quad v^{-q} \tilde{\Gamma}_a v^{-p} = u_a^- \delta^{qp} \quad (b), \quad v_\alpha^{-q} C^{\alpha\beta} v_\beta^{-p} = 0 \quad (c), \quad (3.42)$$

$$2v_\alpha^{+q} v_\beta^{+q} = \Gamma_{\alpha\beta}^a u_a^\# \quad (a), \quad v^{+q} \tilde{\Gamma}_a v^{+p} = u_a^\# \delta^{qp} \quad (b), \quad v_\alpha^{+q} C^{\alpha\beta} v_\beta^{+p} = 0 \quad (c), \quad (3.43)$$

$$2v_{(\alpha}^{-q} \gamma_{qp}^I v_{\beta)}^{+p} = \Gamma_{\alpha\beta}^a u_a^I \quad (a), \quad v^{-q} \tilde{\Gamma}_a v^{+p} = u_a^I \gamma_{qp}^I, \quad I = 1, \dots, 9, \quad (b) \quad (3.44)$$

$$v_\alpha^{+q} C^{\alpha\beta} v_\beta^{-p} = i\delta_{qp} \quad (c).$$

Clearly, the relations (3.42) coincide with (3.24). Notice that just this set of relations, i.e. Eqs. (3.24), are invariant under local  $O(16)$  transformations of  $v_{\alpha q}^-$ ,

$$v_\alpha^{-q} \mapsto v_\alpha^{-p} \mathcal{O}_{pq}, \quad \mathcal{O}_{pp'} \mathcal{O}_{qp'} = \delta_{qp} \quad \Leftrightarrow \quad \mathcal{O}_{qp} \in O(16). \quad (3.45)$$

This symmetry is broken down to  $Spin(9)$  by the constraints (3.44a,b) which involve the  $d = 9$  Dirac matrices

$$\gamma_{qp}^I = \gamma_{pq}^I, \quad (\gamma^I \gamma^J + \gamma^J \gamma^I)_{qp} = \delta^{IJ} \delta_{qp}, \quad q, p = 1, \dots, 16, \quad I = 1, \dots, 9. \quad (3.46)$$

The manifest gauge symmetry of the complete set of constraints (3.42)–(3.44) is  $SO(1,1) \times Spin(9)$ ,

$$v_\alpha^{-q} \mapsto v_\alpha^{-p} S_{pq} e^{-\beta}, \quad v_\alpha^{+q} \mapsto v_\alpha^{+p} S_{pq} e^{+\beta}, \quad (3.47)$$

$$u_a^- \mapsto u_a^- e^{-2\beta}, \quad u_a^\# \mapsto u_a^\# e^{+2\beta}, \quad u_a^I \mapsto u_a^J \mathcal{O}^{JI}, \quad (3.48)$$

where

$$SS^T = I_{16 \times 16}, \quad S_{pp'} \gamma_{p'q'}^I S_{qp'} = \gamma_{qp}^J \mathcal{O}^{JI} \quad \Rightarrow \quad \mathcal{O}^{IK} \mathcal{O}^{JK} = \delta^{IJ},$$

$$\Leftrightarrow \quad S_{qp} \in Spin(9), \quad \mathcal{O}^{IJ} \in SO(9). \quad (3.49)$$

These  $SO(1,1) \times Spin(9)$  transformations also leave invariant the splittings (3.35) of moving frame matrix and (3.34) of the spinor moving frame matrix on rectangular blocks  $v_\alpha^{\pm q}$ . However, if we consider a dynamical model involving only one of these two blocks,  $v_\alpha^{-q}$  in the case of our model, the gauge symmetry is enhanced up to  $[SO(1,1) \otimes SO(9)] \ltimes \mathbb{K}_9$ , where  $\mathbb{K}_9$  transformations are defined by

$$v_\alpha^{-q} \mapsto v_\alpha^{-q}, \quad v_\alpha^{+q} \mapsto v_\alpha^{+q} + v_\alpha^{-p} \gamma_{pq}^I k^{\#I}, \quad (3.50)$$

$$u_a^- \mapsto u_a^-, \quad u_a^\# \mapsto u_a^\# + 2u_a^I k^{\#I} + u_a^- k^{\#I} k^{\#I}, \quad u_a^I \mapsto u_a^I + u_a^- k^{\#I}, \quad (3.51)$$

Thus, in a theory which is invariant under  $SO(1,1) \otimes SO(9)$  transformations (3.47) and does not contain  $v_\alpha^{+q}$ , the set of spinor variables  $v_\alpha^{-q}$  constrained by (3.42) (equivalent to (3.24)) can be identified with homogeneous coordinate of the coset  $SO(1,10)/[SO(1,1) \otimes SO(9)] \ltimes \mathbb{K}_9$  which is isomorphic to a nine-sphere  $\mathbb{S}^9$  [61, 62, 57]

$$\{v_\alpha^{-q}\} = \frac{SO(1,10)}{[SO(1,1) \otimes SO(9)] \ltimes \mathbb{K}_9} = \mathbb{S}^9. \quad (3.52)$$

In the model where these  $v_\alpha^{-q}$  can be treated as spinor moving frame variable, this  $\mathbb{S}^9$  can be recognized as the celestial sphere of the 11D observer [61, 62, 57].

Using the above constraints and their consequences, such as the unity decomposition

$$\delta_\beta^\alpha = iC^{\alpha\gamma}(v_\gamma^{+q}v_\beta^{-q} - v_\beta^{+q}v_\gamma^{-q}) \quad \Leftrightarrow \quad iv_\alpha^{+q}v_\beta^{-q} - iv_\beta^{+q}v_\alpha^{-q} = C_{\alpha\beta}, \quad (3.53)$$

one can check that

$$v^{-q}\tilde{\Gamma}_{ab}v^{-p} := v_\alpha^{-q}\tilde{\Gamma}_{ab}^{\alpha\beta}v_\beta^{-p} = -2i u_{[a}^- u_{b]}^I \gamma^{Iqp}, \quad (3.54)$$

$$v^{-q}\tilde{\Gamma}_{abcde}v^{-p} = -4 u_{[a}^- u_b^I u_c^J u_d^K u_{e]}^L \gamma^{IJKLqp}. \quad (3.55)$$

For the generalized superparticle model (3.30) the canonical momenta conjugate to the tensorial coordinate functions  $y^{ab}$  and  $y^{abcde}$  have the form of (3.22) with composite bosonic spinor (3.23), so that (3.54) implies

$$p_{ab} = 2 u_{[a}^- u_{b]}^I \lambda^+ \gamma^I \lambda^+, \quad (3.56)$$

$$p_{abcde} = -4 u_{[a}^- u_b^I u_c^J u_d^K u_{e]}^L \lambda^+ \gamma^{IJKL} \lambda^+, \quad (3.57)$$

where  $\lambda^+ \gamma^I \lambda^+ := \lambda_q^+ \gamma^{Iqp} \lambda_p^+$ , etc. This set of equations has the form of the general solution (3.28) of the classical section conditions (3.10), (3.11) with

$$q_a = u_a^I \frac{\lambda^+ \gamma^I \lambda^+}{(\lambda^+ \lambda^+)}, \quad q_{abcd} = -4 u_a^I u_b^J u_c^K u_d^L \frac{\lambda^+ \gamma^{IJKL} \lambda^+}{(\lambda^+ \lambda^+)}. \quad (3.58)$$

Hence we have shown that the preonic superparticle with composite bosonic spinor (3.23), described by the action (3.30), generates a solution of the classical counterparts (3.10), (3.11) of the proposed section conditions (3.9) of the hypothetical uEFT. In this sense we can say that (3.30) is (one of the) *uEFT superparticle(s)*.

### 3.3.4 A family of superparticle 'solving' the classical section conditions

The next natural question is: are there more uEFT superparticle models? In this section we present the family of superparticle models in  $\Sigma^{(528|32)}$  superspace, first described in [72], and show that each of these generates a constraint solving the classical section conditions (3.10), (3.11) of the hypothetical 11D EFT. The actions of these models can be collected in the universal expression

$$S = \int d\tau \rho_{qp}^{\#} v_{\alpha}^{-q} v_{\beta}^{-p} \Pi_{\tau}^{\alpha\beta} \equiv \int d\tau \rho_{qp}^{\#} v_{\alpha}^{-q} v_{\beta}^{-p} (\partial_{\tau} X^{\alpha\beta} - i \partial_{\tau} \theta^{(\alpha} \theta^{\beta)}) , \quad (3.59)$$

in which  $\rho_{qp}^{\#} = \rho_{qp}^{\#}(\tau)$  is a symmetric  $16 \times 16$  bosonic matrix field,  $X^{\alpha\beta} = X^{\alpha\beta}(\tau)$  and  $\theta^{\alpha} = \theta^{\alpha}(\tau)$  are 528 bosonic and 32 fermionic coordinate functions, the same as in (3.17) and (3.30), and  $v_{\alpha}^{-q} = v_{\alpha}^{-q}(\tau)$  are the spinor moving frame variables (3.52) discussed in the sec. 3.3.3.

One can consider the action (3.59) as describing a class of superparticle models the properties of which depend essentially on the rank of symmetric matrix  $\rho_{qp}^{\#}$ . Alternatively one can speak about dynamical system with several branches determined by this rank. Of these, let us especially notice the following particular cases preserving minimal and maximal amount of supersymmetry:

- The case of rank 16 matrix with unity eigenvalues,

$$\rho_{pq}^{\#} = \rho^{\#} \delta_{pq} , \quad (3.60)$$

describes the massless 11D superparticle (sometimes called M0-brane), see [51, 52]. This model has 16  $\kappa$ -symmetries and, correspondingly, its ground state preserves one half of 32 spacetime supersymmetries.

- The case of rank 1 matrix

$$\rho_{pq}^{\#} = \lambda_q^{+} \lambda_p^{+} , \quad (3.61)$$

as discussed below, correspond to a preonic superparticle model (in terminology of [73]). It possesses 31  $\kappa$ -symmetries and, hence, its ground state preserves all but one supersymmetries.

Generically, if we restrict the model by requiring all the eigenvalues of matrix  $\rho_{pq}^{\#}$  of the rank  $r$  to be positive, it always can be written in the form

$$\rho_{pq}^{\#} = \lambda_q^{+s} \lambda_p^{+s} , \quad s = 1, \dots, r . \quad (3.62)$$

Thus, without loss of generality (in practical terms, i.e. if not considering a problematic models) one can describe the branch of the dynamical system (3.59) with  $\text{rank}(\rho_{pq}^{\#}) = r$  by

$$S^{(r)} = \int d\tau \lambda_q^{+s} \lambda_p^{+s} v_{\alpha}^{-q} v_{\beta}^{-p} \Pi_{\tau}^{\alpha\beta} , \quad s = 1, \dots, r . \quad (3.63)$$

In this family  $S^{(1)}$  is the preonic action, corresponding to (3.61), while the standard massless superparticle action is  $S^{(16)}$  with  $\lambda_q^{+s} = \sqrt{\rho^\#} \delta_q^s$ .

The action (3.63) is invariant under the (32- $r$ )-parametric local fermionic  $\kappa$ -symmetry

$$\begin{aligned} \delta_\kappa X^{\alpha\beta} &= i\delta_\kappa \theta^{(\alpha} \theta^{\beta)} , & \delta_\kappa v_\alpha^{-q} &= 0 , & \delta_\kappa \lambda_q^{+s} &= 0 \\ \delta_\kappa \theta^\alpha &= \kappa^{+q} v_q^{-\alpha} + \kappa^{-\tilde{s}} w_q^{\tilde{s}} v_q^{+\alpha} , & \tilde{s} &= 1, \dots, (16-r) . \end{aligned} \quad (3.64)$$

Notice that Eqs. (3.64) describes the general solution of the equation

$$\delta_\kappa \theta^\alpha v_\alpha^{-q} \lambda_q^{+s} = 0, \quad \left\{ \begin{array}{l} q = 1, \dots, 16 , \\ s = 1, \dots, r \end{array} \right\} \quad \Leftrightarrow \quad \delta_\kappa \theta^\alpha v_\alpha^{-q} \rho_{qp}^\# = 0, \quad \text{rank}(\rho_{qp}^\#) = r . \quad (3.65)$$

To write this solution we have introduced the set of Spin(1,10) spinors  $v_q^{\pm\alpha} = \pm i C^{\alpha\gamma} v_\gamma^{\pm q}$  which obey (see (3.53))

$$v_q^{+\alpha} v_\alpha^{-q} = \delta_{pq} , \quad v_q^{-\alpha} v_\alpha^{-q} = 0 , \quad (3.66)$$

and a set of 16-vectors  $w_q^{\tilde{s}}$  orthogonal to  $\lambda_q^{+s}$

$$w_q^{\tilde{s}} \lambda_q^{+s} = 0 , \quad , \quad s = 1, \dots, r , \quad \tilde{s} = 1, \dots, (16-r) . \quad (3.67)$$

In other words, that are (16- $r$ ) null-vectors of the rank  $r$  matrix  $\rho_{qp}^\# = \lambda_q^{+s} \lambda_p^{+s}$ ,  $w_q^{\tilde{s}} \rho_{qp}^\# = 0$ .

Let us calculate the canonical momentum conjugate to the bosonic coordinates in (3.59). In the spin-tensor notation we obtain

$$p_{\alpha\beta} = \rho_{qp}^\# v_\alpha^{-q} v_\beta^{-p} .$$

Using the constraints (3.24) we can find that this implies that the spacetime momentum of the system is a light-like 11-vector

$$p_a = \frac{\rho_{qq}}{32} u_a^- \quad \Rightarrow \quad p_a p^a = 0 .$$

Hence from the 11D spacetime perspective, any of the models (3.63) describes a massless particle or a set of massless particles.

Furthermore, using (3.54) it is not difficult to show that the momenta conjugate to the tensorial coordinates have the form

$$p_{ab} = u_{[a}^- q_{b]}^\# \quad \text{and} \quad p_{abcde} = u_{[a}^- q_{bcde]}^\#$$

with

$$q_a = u_a^I \frac{\lambda^{+r} \gamma^I \lambda^{+r}}{(\lambda^{+s} \lambda^{+s})} , \quad q_{abcd} = -4 u_a^I u_b^J u_c^K u_d^L \frac{\lambda^{+r} \gamma^{IJKL} \lambda^{+r}}{(\lambda^{+s} \lambda^{+s})} . \quad (3.68)$$

Thus any model from the family described by a (nondegenerate) action of the form (3.59) or (3.63) generate a solution of the classical section conditions (3.10), (3.11) of the hypothetical underlining uEFT.

## 4. On uEFT superparticles and 11D higher spin theories

In the previous Section 3.3 we have presented a family of superparticle models which produce as constraints quite generic solutions of the section conditions proposed for the hypothetical underlying 11D EFT (uEFT) in Sec. 3.2. In this section we will argue that, curiously enough, the quantization of these uEFT superparticles should result in the theory of free massless higher spin fields in 11 dimensional spacetime.

### 4.1 Free $D = 4, 6, 10$ conformal higher spin theory description in $\Sigma^{(\frac{m(m+1)}{2}|m)}$ superspace with $m = 2(D - 2) = 4, 8, 16$

To agree in favor of the above conclusion, we begin with already mentioned relation of the original preonic superparticle model (3.17) with  $m = 4, 8$  and  $16$  ( $\alpha, \beta = 1, \dots, m$ ) with free conformal massless higher spin field theories in spacetime of dimensions  $D = \frac{m+4}{2} = 4, 6, 10$  [86, 92]. Namely, the quantization of these models of superparticle in  $\Sigma^{(\frac{m(m+1)}{2}|m)}$  superspace with  $m = 2(D - 2) = 4, 8, 16$  results in the quantum state spectrum described by an infinite tower of all  $D=4, 6$  and  $10$  massless conformal higher spin fields.

This is related to the fact that generalized superconformal symmetry  $OSp(1|2m)$  can be realized on towers of the bosonic and of the fermionic massless conformal fields which can be packed into a scalar  $\phi(X)$  and a 'spinor' (s-vector) field  $f_\alpha(X)$  on the tensorial space (hyperspace)  $\Sigma^{(\frac{m(m+1)}{2}|0)}$  (see [97] for  $m = 4$ ) which obey the Vasiliev's equations  $\partial_{\alpha[\beta}\partial_{\gamma]\delta}\phi(X) = 0$  (3.20) and  $\partial_{\alpha[\beta}f_{\gamma]}(X) = 0$  [87, 88]. These fields can be also collected in superfield defined on  $\Sigma^{(\frac{m(m+1)}{2}|m)}$  superspace satisfying  $D_{[\alpha}D_{\beta]}\Phi(X, \theta) = 0$  [91].

On the other hand, all the tower of the solutions of all the free conformal higher spin equations for bosonic fields in  $D=4, 6$  and  $10$  can be described by a scalar function  $\tilde{\phi}(\lambda)$  of one unconstrained real bosonic spinor  $\lambda_\alpha$ ,  $\alpha = 1, \dots, m$ , with  $m = 2(D - 2)$ , subject to the restriction to be even with respect to  $\lambda_\alpha \rightarrow -\lambda_\alpha$ , and also by specifying the class of functions  $\tilde{\phi}(\lambda)$  belongs to [86, 92]. With a suitable choice of this latter the solution of the bosonic Vasiliev equation (3.20) is given by

$$\phi(X) = \int d\lambda \tilde{\phi}(X, \lambda) = \int d\lambda \tilde{\phi}(\lambda) e^{i\lambda_\alpha \lambda_\beta X^{\alpha\beta}}. \quad (4.1)$$

### 4.2 $m=4, 8, 10$ counterparts of the preonic superparticle and conformal higher spin fields in $D=4, 6, 10$

This  $\tilde{\phi}(\lambda)$ , and also its fermionic counterpart, can be obtained by quantization [86, 92] of the  $m = 4, 8, 10$  versions of the superparticle model (3.17) in terms of components of orthosymplectic twistor  $(\lambda_\alpha, \mu^\alpha, \eta)$  related to the  $\Sigma^{(\frac{n(n+1)}{2}|n)}$  coordinates by

$$\mu^\alpha = X^{\alpha\beta} \lambda_\beta - \frac{i}{2} \theta^\alpha \theta^\beta \lambda_\beta, \quad \eta = \theta^\alpha \lambda_\alpha. \quad (4.2)$$

The fundamental representation of  $OSp(1|2m)$  acts on orthosymplectic supertwistors by left multiplication, and the above incidence relations (4.2) explain the possibility to realize  $OSp(1|2m)$  as superconformal symmetry of  $\Sigma^{(\frac{m(m+1)}{2}|m)}$ .

For simplicity, we restrict our discussion here by quantization of purely bosonic limit,  $\theta = 0$ , of  $m = 2(D - 2) = 4, 8, 16$  superparticle (3.17). Using the Leibniz rule the bosonic action in (3.17),

$$S_0 = \int d\tau \lambda_\alpha \lambda_\beta \partial_\tau X^{\alpha\beta} , \quad (4.3)$$

can be written in the form

$$S = \int d\tau (\lambda_\alpha \partial_\tau \mu^\alpha - \partial_\tau \lambda_\alpha \mu^\alpha) \quad (4.4)$$

with  $\mu^\alpha = X^{\alpha\beta} \lambda_\beta$  (4.2). This new variable  $\mu^\alpha(\tau)$  carries all the physical degrees of freedom in  $X^{\alpha\beta}(\tau) = X^{\beta\alpha}(\tau)$  (the remaining  $\frac{m(m-1)}{2}$  components can be gauged away) and can be considered as a momentum conjugate to  $\lambda_\alpha$  (or *vice versa*: coordinate conjugate to momentum  $\lambda_\alpha$ ) and the action (4.4) can be considered as a Hamiltonian action with Hamiltonian equal to zero.

Then the quantization of the model (4.4) is trivial; its state vector can be represented by an arbitrary function of  $\lambda_\alpha$ ,  $\tilde{\phi}(\lambda)$ . The spacetime treatment of this quantum state spectrum uses the relation (3.18),

$$p_{\alpha\beta} - \lambda_\alpha \lambda_\beta = 0 , \quad (4.5)$$

which can be obtained as a primary constraint when constructing Hamiltonian approach to our dynamical system on the basis of the original action (4.3).

An alternative quantization of (4.3) with  $m = 2(D - 2) = 4, 8, 16$ , which passes through the stage of development of such a Hamiltonian approach and conversion of the second class constraints [86], results in a wavefunction dependent on both  $X^{\alpha\beta}$  and  $\lambda_\gamma$  and obeying the quantum counterpart of the constraint (4.5), the so-called preonic equation

$$(\partial_{\alpha\beta} + i\lambda_\alpha \lambda_\beta) \varphi(X, \lambda) = 0 . \quad (4.6)$$

The solution of this equation is given by  $\tilde{\phi}(X, \lambda) = \tilde{\phi}(\lambda) e^{i\lambda_\alpha \lambda_\beta X^{\alpha\beta}}$  and its integration with a suitable measure  $d^n \lambda$  give the wavefunction in the generalized coordinate,  $X^{\alpha\beta}$  representation (4.1).

On the other hand, the wavefunction in the momentum representation,  $\phi(p_{\alpha\beta})$ , is localized on the solutions of (4.5), which implies, in particular, that the standard  $D$ -vector momentum extracted from (4.5) is

$$p_a = \lambda \tilde{\Gamma}_a \lambda \equiv \lambda_\alpha \tilde{\Gamma}_a^{\alpha\beta} \lambda_\beta . \quad (4.7)$$

For  $D = 4, 6, 10$  (and also for  $D = 3$ ) this momentum is light-like  $p_a p^a = 0$  and hence the quantum states of the model are massless.

Actually, the quantum state spectrum of  $D=4$  model consists of an infinite tower of the massless fields of all possible helicities. In the case of  $D=6$  and  $D=10$  model, where, in contrast to  $D = 4$ , not all the free massless fields are conformal, we obtain a tower of massless conformal higher spin fields (see e.g. [92] for their description). The fields in the

tower are 'enumerated' by a set of integer numbers which can be considered as momenta conjugate to the coordinates of  $\mathbb{S}^{(D-3)}$  ( $\mathbb{S}^1$ ,  $\mathbb{S}^3$  and  $\mathbb{S}^7$ ) spheres realized as Hopf fibrations

$$\mathbb{S}^{(n-1)}/\mathbb{S}^{D-2} = \mathbb{S}^{2D-5}/\mathbb{S}^{D-2} = (\mathbb{S}^3/\mathbb{S}^2, \mathbb{S}^7/\mathbb{S}^4, \mathbb{S}^{15}/\mathbb{S}^8) .$$

Let us describe how these appear. The space of light-like momenta in D-dimensions is

$$\{p_a | p^2 = 0\} = \mathbb{R}_+ \otimes \mathbb{S}^{(D-2)} \quad (4.8)$$

The space of nonvanishing  $n$ -component bosonic spinors our wavefunction  $\tilde{\phi}(\lambda)$  depends on is

$$\{\lambda_\alpha\} = \mathbb{R}^n - \{0\} = \mathbb{R}_+ \otimes \mathbb{S}^{(n-1)} , \quad (4.9)$$

and the scale of momenta ( $\mathbb{R}_+$  in (4.8)) is given by the square of the scale of the bosonic spinor ( $\mathbb{R}_+$  in (4.9)). Thus, besides the light-like momenta, the wavefunction depend on  $(D-3)$  coordinates of the fibrations

$$\{\lambda_\alpha\}/\{p_a | p^2 = 0\} = \mathbb{S}^{(n-1)}/\mathbb{S}^{(D-2)} = \mathbb{S}^{2D-5}/\mathbb{S}^{(D-2)} , \quad D = 4, 6, 10 , \quad (4.10)$$

which are isomorphic to  $\mathbb{S}^{(D-3)}$  spheres. These spaces are compact and a momentum conjugate to a compact coordinate is quantized.

Hence passing to the momentum representation on this compact directions, we will arrive at the wave function depending on D dimensional light-like momentum ( $D = 4, 6, 10$ ) and characterized by  $(D-3)$  integer numbers. In D=4 one integer number obtained in such a way is the doubled helicity of a massless fields. The description of D=10 and D=6 conformal higher spin fields can be found in [92] and refs. therein.

#### 4.3 On superparticle models for massless non-conformal higher spin theories in D=6,10 and D=11

Notice that, although the interest in a tensorial (super)space or hyper(super)space description of higher spin fields persists already more than 15 years [86, 87, 88, 89, 90, 91, 92, 93, 94, 95], the research is mainly concentrated on D=4 case. The reason beyond this, besides that the  $m = 8, 16$  cases are more complicated, is that, in contradistinction to D=4, in D=6 and D=10 dimensional cases not all the massless fields are conformal, and these which are look quite exotic [92]. In particular neither the linearized equations for graviton, nor the Maxwell equations for D-vector potential are conformally invariant in D=6 and D= 10 dimensions.

In the 11D case the straightforward generalization of the above derivation of free higher spin theories fails. Namely, symplectic twistor quantization is universal and results in a wavefunction  $\tilde{\phi}(\lambda)$  depending on  $m = 32$  component bosonic spinor in an arbitrary manner, but what fails is its spacetime interpretation: in contrast to D=4,6,10 cases, in D=11 the momentum constructed from spinor bilinear (3.18) is not light-like. Actually with this 11-momentum the mass of the quantum states remains indefinite which hamper the spacetime interpretation of D=11 (super)particle model (4.3) ((3.17)).

In the above perspective there exist the quests for superparticle models providing the classical mechanic description of D=11 higher spin field theory and of D=6, 10 dimensional massless *non-conformal* higher spin theories. The  $m = 32$  and  $m = 8, 16$  versions of the above described superparticle models (3.59) and (3.63) are good candidates for these roles.

This conjecture is suggested by a series of observations the first of which is that, as we have described above, all these models produce the constraints  $p_a \propto u_a^-$  which implies  $p_a p^a = 0$ . As a result, their quantum state spectrum is formed by massless states. Then, the analogy with the above discussed  $n = 2(D - 2) = 4, 8, 16$  version of the preonic superparticle model suggests that this quantum state spectrum provides us with a theory of free higher spin fields.

This conjecture looks especially natural in the case of preonic-type model (3.30) with composite spinor field (3.23). As far as the other models (3.59), (3.63), preserving from one half to all but two supersymmetries are concerned, this might be considered as a counterpart of the D=4  $OSp(4|2)$  invariant models in [86] which preserve 2 of 4 supersymmetries and also describes supermultiplet of free massless higher spin fields by its quantum state spectrum.

Furthermore, as there are no traces of conformal invariance in the models (3.59), (3.63) with  $m = 8, 16, 32$ , their quantum state spectrum should not be conformal. Indeed, it is easy to see that even in the preonic-type model with composite bosonic spinor (3.30), which is included as  $r = 1$  representative in the set of models (3.63), the presence of spinor moving frame variables  $v_{\alpha q}^-$  in (3.23) breaks the  $Sp(m)$  invariance down to D-dimensional Lorentz group.

Thus we have argued that the quantization of the models (3.63), (3.59) with  $m = 8, 16$  and 32 should result in a theory of free non-conformal higher spin fields in  $D = 6, 10$  and  $D = 11$  dimensional spacetime.

The check of this conjecture by explicit quantization of these models and by the analysis of their quantum state spectrum will be the subject of a forthcoming paper. Here we just notice that the idea on that the 11D higher spin fields as necessary ingredients of (the hypothetical) underlying 11D exceptional field theory, uEFT, is in consonance with discussions on their necessity in the context of (also hypothetical)  $E_{10}$  and  $E_{11}$  theories [43, 98].

## 5. Conclusions and discussion

In this paper we conjectured the existence of hypothetical underlying exceptional field theory, which we abbreviated as 11D EFT of uEFT, defined in the maximal tensorial central charge superspace

$$\Sigma^{(528|32)} = \{x^a, y^{ab}, y^{abcde}, \theta^\alpha\}, \quad a, b, c, d, e = 0, 1, \dots, 9, 10, \quad \alpha = 1, \dots, 32, \quad (5.1)$$

which is the group manifold associated to the M-theory superalgebra (3.1). We have presented some arguments in favor of this conjecture, based on the hypothesis that the additional coordinates of all the  $E_{n(n)}$  EFTs with  $n \leq 8$  should be related to the maximally



extended  $d = 11 - n$  dimensional supersymmetry algebra, and have proposed the candidate section conditions for this hypothetical uEFT

$$\partial_{[a} \otimes \partial_{bc]} + \partial_{[bc} \otimes \partial_a] = 0, \quad \partial_{[a} \otimes \partial_{bcdef]} + \partial_{[bcdef} \otimes \partial_a] = 0, \quad (5.2)$$

$$\begin{aligned} \partial_{[ab} \otimes \partial_{bc]} &= 0, & \partial_{[a} \otimes \partial_{bcdef]} + \partial_{[bcdef} \otimes \partial_a] &= 0, \\ \partial_{[ab} \otimes \partial_{cdefg]} + \partial_{[cdefg} \otimes \partial_{ab]} &= 0. \end{aligned} \quad (5.3)$$

To check that these section condition are reasonable, i.e. that their general solution is not trivial, we have discussed a series of superparticle models in  $\Sigma^{(528|32)}$  and show that they produce quite generic solutions of (the classical counterparts of) these hypothetical section conditions as constraints on their generalized momenta. Of course, the next question was what is the physical meaning of these superparticle models. To address it we have presented some arguments that these superparticle models should produce free massless 11D higher spin field theories as their quantum state spectrum.

By passing, we have also argued that 10D ( $m = 16$ ) and 6D ( $m = 8$ ) counterparts of these superparticle models, defined in  $\Sigma^{\left(\frac{m(m+1)}{2}|m\right)}$  superspaces with  $m = 2(D-2)$ , provides a classical mechanics description of free non-conformal massless higher spin theories in  $D=10$  and  $D=6$ . These are of interest because the most interesting massless fields, like  $D$ -dimensional graviton and photon, are not conformal in  $D \neq 4$ .

The above observations suggest that the hypothetical underlying uEFT or 11D EFT should contain the 11d higher spin theory as an important sector. Interestingly enough, 11D Higher spin fields were recently considered as a probably necessary ingredients of completion of 11D supergravity till a  $E_{10}$  or even  $E_{11}$  invariant theories [98].

Even before, the relation of  $E_{11}$  with higher spin theories was discussed in [43] where the action of [71] supplemented by the condition of light-likeness of the bilinear  $\lambda \Gamma_a \lambda$ , is proposed as a candidate for low level (three level) approximation of a hypothetical point particle model based on a non-linear realization of  $E_{11} \ltimes l_1$ . Here  $l_1$  is the fundamental representation associated with the 'far end' of the  $E_{11}$  Dynkin diagram, usually called 'node 1' (see e.g. [44]), the (infinite) set generators of which contains, at lowest level,  $P_a$ ,  $Z^{ab}$ ,  $Z^{abcde}$ , which can be identified with the bosonic generators of M-algebra (3.1).

Such a three level approximation to hypothetical  $E_{11}$  superparticle can be identified with the 'preonic' representative (3.30) of the family of the action of uEFT superparticles (3.63), (3.59). It would be interesting to understand a possible role of other representatives of this family in an  $E_{11}$  perspective.

Thus 11D higher spin theories are probably common ingredients of the uEFT and of the  $E_{11}$  and  $E_{10}$  theories. On the other hand, these are not identical, but rather complementary. As it is seen in Table 1,  $E_{n(n)}$  EFTs, making manifest the the rigid  $E_{n(n)}$  duality symmetry, keep only the  $SO(1, 10 - n)$  subgroup of the 11D  $SO(1, 10)$  Lorentz group manifest. A part of Lorentz symmetry is sailed for an increase of the manifest rigid symmetry<sup>10</sup>. In this consequence  $E_{10}$  and  $E_{11}$  clearly correspond to  $d = 1$  and  $d = 0$  (!), thus

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<sup>10</sup>As  $E_{n(n)}$  EFT is an extension of  $d$  dimensional supergravity, the Lorentz  $SO(1, d - 1)$  symmetry is one of its manifest gauge symmetries while  $E_{n(n)}$  is the rigid symmetry. Although our discussion here used the

not leaving any part of the  $SO(1,10)$  Lorentz symmetry manifest, while our uEFT correspond to  $D=11$  and  $n=0$ , thus leaving all the Lorentz symmetry but no rigid symmetry manifest. Together with the Lorentz symmetry this hypothetical underlying theory should possess manifest supersymmetry the generators of which form the M-theory superalgebra, the maximally enlarged supersymmetry algebra in 11D. The uEFT is defined in the 11D superspace enlarged by coordinate conjugate to the tensorial central charge generators of this superalgebra, (5.1), which provides the resource for all the additional coordinates of the lower  $d$ /higher  $n$  EFTs with  $n < 9$ .

As a potentially suggestive speculation, let us to try to incorporate the hypothetical  $E_{10}$  and  $E_{11}$  theories and uEFT in the Table 1. Clearly, the first two should be put on the top and the last - at the bottom of the Table 1, see Table 2 below.

If doing just this, we would have the holes at  $n=9$  ( $d=2$ ) and  $n=1$  ( $d=10$ ). The first of this positions should clearly correspond to some hypothetical  $E_9$  EFT having manifest symmetry under the infinite dimensional Katz-Moody algebra  $E_9$  [77]. The second,  $n=1$  hole we have filled in Table 2 by putting the doubled field theory, DFT, in this line. The main motivation is that this case clearly correspond to  $d=10$ , so that its association with  $n=1$  comes just from  $n=11-d$ . The increasing of the number of additional coordinates till 10, in contrast to their decreasing from 248 to 3 when  $d$  increased from 3 to 9, might mark the change of tendency which is then continuing on the stage of passing to 11D uEFT, which is defined in (super)space with  $517=528-11$  additional coordinates. Our hypothesis does not associates the T-duality group  $O(10,10)$  with a hypothetical  $E_{1(1)}$  as far as the 10d Lorentz group  $SO(1,9)$  is a subgroup of  $O(10,10)$ . Rather  $E_{1(1)}$  should be associated with the generators of the coset  $O(10,10)/SO(1,9)$  so that, although a big number of duality symmetries is present, they do not form a closed algebraic structure themselves, but together with Lorentz group only.

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models in flat superspaces where the  $SO(1,D-1)$  is not local, our terminology in this section is borrowed from the complete description of EFTs.

$E_{n(n)}$ of the EFT	n	d=11-n	$N_n = \#$ of $y^\Sigma$	Section condition
? $E_{11}$	11	d=0	$\infty?$	?
? $E_{10}$	10	d=1	$\infty?$	?
? $E_9$	9	d=2	$\infty?$	?
$E_{8(8)}$	8	d=3	248	$Y_{\Lambda\Xi}^{\Sigma\Pi} \partial_\Sigma \otimes \partial_\Pi = 0$ , [10]
$E_{7(7)}$	7	d=4	56	$t_G^{\Sigma\Pi} \partial_\Sigma \otimes \partial_\Pi = 0$ , [9]
$E_{6(6)}$	6	d=5	27	$d^{\Lambda\Sigma\Pi} \partial_\Sigma \otimes \partial_\Pi = 0$ [7]
$E_{5(5)} = SO(5, 5)$	5	d=6	16	$\gamma_I^{\Sigma\Pi} \partial_\Sigma \otimes \partial_\Pi = 0$ [15]
$E_{4(4)} = SL(5)$	4	d=7	10 ( $y^{ab} = y^{[ab]}$ )	$\partial_{[ab} \otimes \partial_{cd]} = 0$ [3, 6]
$E_{3(3)} = SL(3) \times SL(2)$	3	d=8	6 ( $y^{\alpha i}$ )	$\epsilon^{ijk} \epsilon^{\alpha\beta} \partial_{\alpha i} \otimes \partial_{\beta j} = 0$ [14]
$E_{2(2)} = SL(2) \times \mathbb{R}^+$	2	d=9	3 ( $y^\alpha, z$ )	$\partial_z \otimes \partial_\alpha + \partial_\alpha \otimes \partial_z = 0$ [18]
DFT: $SO(10, 10)$	1?	d=10	10 ( $\tilde{x}_\mu$ )	$\partial_\mu \otimes \tilde{\partial}^\mu + \tilde{\partial}^\mu \otimes \partial_\mu = 0$ [22, 23]
<b>uEFT</b> : only $SO(1, 10)$ is manifest	0?	d=11	528 ( $y^{ab}, y^{abcde}$ )	$\partial_{[a} \otimes \partial_{bc]} + \partial_{[bc} \otimes \partial_a] = 0$ , $\partial_{[a} \otimes \partial_{bcdef]} + \partial_{[bcdef} \otimes \partial_a] = 0$ , $\partial_{[ab} \otimes \partial_{bc]} = 0$ , <i>etc.</i> (see (5.3))

Table 2. Known and hypothetical EFTs. Conjectured place of underlying EFT and hypothetical EFTs for infinite dimensional groups in Table 1.

The above discussion suggests the origin of doubled spacetime coordinate of DFT in the uEFT superspace  $\Sigma^{(528|32)}$ , and furthermore, that our 11D uEFT provides a unification of DFT with  $n = 2, \dots, 8$   $E_{n(n)}$  exceptional field theories.

We hope that our underlying 11D exceptional field theory (11D uEFT) conjecture will be useful for deeper understanding of dualities and of the structure of/beyond the M-theory. It will be interesting to elaborate further on its interrelation/complementary with the  $E_{11}$  and  $E_{10}$  proposal. One of the important directions of further development of our approach is related to the quantization of the family of the generalized superparticle models (3.63). In particular this should provide us with a description of towers of massless non-conformal higher spin theories in D=11, the possible fundamental role of which was also a subject of thinking in the Higher Spin community [99].

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